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Groups Isomorphic via Decomposition

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In *Advances in Applied Mathematics* **8** (1987), 281–344, Problem III.2 [1], R. Daniel Mauldin and S. M. Ulam have proposed the problem:

> Are there non-isomorphic groups G and H such that G and H can be decomposed into two parts: $G = G_1 \cup G_2$ and $H = H_1 \cup H_2$ with $G_1 \sim H_1$ and $G_2 \sim H_2$. The notation $A \sim B$, where $A \subset G$ and $B \subset H$, means there is a one to one map f of A onto B such that if x, y, and xy are all in A, then $f(xy) = f(x)f(y)$.

The purpose of this note is to give an elementary set of examples for groups G and H.

Let G be any group with a normal subgroup N of index 2. Now $G = N \cup gN$ for any $g \notin N$, and $N = N \cup N$. Clearly $N \sim N$ via the identity map. We also have $gN \sim N$ via the map f: $gN \to N$ defined by $f(gn) = n$. f is obviously one to one and onto. Now G/N is a cyclic group of order two, and thus $(gN)(gN) = N$, and so if x and y are elements of gN then $xy \in N$. Thus the condition $f(xy) = f(x)f(y)$ if x, y, and xy are all in gN, is vacuously satisfied.

The answer to the question is affirmative and just as easy if we further require both decompositions to be partitions. To construct examples pick any group N such that you can find two non-isomorphic extensions of N by a cyclic group of order two. Again these two extensions will work as examples of G and H. The partitions into the cosets of N will be the required decompositions. A specific example of this is Q_8 and D_8, the quaternions and the dihedral group of order 8.

Of course, in both cases by changing the index $|G : N|$, we can find examples of groups "isomorphic via decomposition" into $|G : N|$ parts.

REFERENCE

1. R. D. MAULDIN AND S. M. ULAM, Mathematical problems and games, *Adv. in Appl. Math.* **8** (1987), 281–344.

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